

ST 563: Theory of Statistics III

Week 7 Recitation

1. (9.2.2 from *Probability and Statistical Inference*) Let X be a random variable from the distribution with pdf

$$f(x|\theta) = \frac{2(\theta - x)}{\theta^2} \mathbf{1}\{0 < x < \theta\}$$

where $\theta > 0$ is the unknown parameter of interest.

- (a) Show that X/θ is a pivot, and find its distribution.
(b) Derive a $(1 - \alpha)100\%$ confidence interval for θ
2. (9.26 from *Statistical Inference, 2nd Edition*) Let X_1, \dots, X_n be i.i.d observations from a $\text{Beta}(\theta, 1)$ pdf

$$f(x|\theta) = \theta x^{\theta-1}, \quad 0 < x < 1, \theta > 0$$

and assume that θ has a $\text{Gamma}(r, \lambda)$ prior

$$\pi(\theta|r, \lambda) = \frac{\lambda^r}{\Gamma(r)} \theta^{r-1} e^{-\lambda\theta}, \quad \theta > 0.$$

Find a $1 - \alpha$ Bayes credible set for θ .